MINISTRY OF EDUCATION & TRAINING MINISTRY OF NATIONAL DEFENSE

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Le Pham Binh

LINEAR STATIC AND DYNAMIC ANALYSIS OF FGM POROUS NANOPLATES RESTING ON ELASTIC FOUNDATION USING NONLOCAL ELASTICITY THEORY

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Supervisors:

1. **Assoc. Prof. Dr. Doan Trac Luat**
2. **Assoc. Prof. Dr. Le Minh Thai**

Opponent 1: Prof. Dr.Sc. Nguyen Tien Khiem
Vietnam Academy of Science and Technology

Opponent 2: Prof. Dr.Sc. Nguyen Dinh Duc
Vietnam National University, Hanoi

Opponent 3: Prof. Dr. Tran Van Lien
Hanoi University of Civil Engineering

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LIST OF PUBLICATIONS

1. Trac Luat Doan, **Pham Binh Le**, Trung Thanh Tran, Vu Khac Trai, Quoc Hoa Pham (2021). *Free Vibration Analysis of Functionally Graded Porous Nano-plates with Different Shapes Resting on Elastic Foundation*. Journal of Applied and Computational Mechanics, Vol. 7, Issue 3. doi.org/10.22055/JACM.2021.36181.2807, pp. 1593-1605 (SCOPUS).

2. **Le Pham Binh**, Doan Trac Luat, Le Minh Thai, Tran Trung Thanh (2021). *Static bending analysis of annular nanoplates resting on elastic foundation using nonlocal elasticity theory*. Journal of Science and Technique – Le Quy Don Technical University, Vol. 16, No. 02, pp. 23-32.

3. **Le Pham Binh**, Doan Trac Luat, Tran Trung Thanh, Pham Quoc Hoa and Pham Tien Dat, (2021). *Forced vibration of FGP nanoplates resting on elastic foundation using finite element formulation*. Proceedings of The 15th National Conference on Solid Mechanics, Thai Nguyen – 2021, pp. 81-90.

4. **Le Pham Binh**, Le Minh Thai, Pham Tien Dat and Tran Trung Thanh, (2021). *Static behavior of FGP half-annular nanoplates resting on elastic foundation using nonlocal elasticity theory*. Proceedings of The 15th National Conference on Solid Mechanics, Thai Nguyen – 2021, pp. 91-99.

5. Tran Trung Thanh, **Le Pham Binh** (2022). *Nonlocal Dynamic Response Analysis of Functionally Graded Porous L-shape Nanoplates Resting on Elastic Foundation Using Finite Element Formulation*. Engineering with Computer. https://doi.org/10.1007/s00366-022-01679-6, (ISI, Q1).

INTRODUCTION

1. Motivation

Due to their special mechanical, thermal, electrical, and chemical properties, nanostructures are increasingly used in the fields of medicine, electronics, and so on. Nanoplates are one of the most important structures commonly used as components in thin films, resonators, and sensors. Therefore, studying the vibrations of nanostructures is very important for design and manufacturing.

With nanostructures, the dimensional effect becomes special. The test and simulation results show a significant influence on the mechanical properties when the size of the structure becomes small. When the length of the plate decreases, the influence of the intermolecular forces on the static and dynamic properties is significant and cannot be neglected. Since nano experiments are difficult and the simulation of molecular dynamics is expensive, the development of mathematical models at the nanoscale has become a key issue for evaluating the mechanical behavior of nanostructures.

So the topic ***"Linear Static and Dynamic Analysis of FGM Porous Nanoplates Resting on Elastic Foundation Using Nonlocal Elasticity Theory"*** is an urgent topic that has scientific and practical significance.

2. Aim of this thesis

In general, studying nanostructures and the mechanical behavior of nanostructures is a large field. Within the framework of the thesis, the author focuses on studying the static and dynamic response of the functionally graded material (FGP) nanoplates using the finite element method (FEM) based on nonlocal elasticity theory (NET) with the following specific objectives:

- Establishing governing equations and algorithms to calculate the stress, strain, and displacement of the FGP nanoplates subjected to static and dynamic loads.

- The calculation programs are established by the Matlab software to analyze displacement, stress, free vibration, and dynamic response of FGP nanoplates resting on an elastic foundation.

- Investigating the influence of some structural factors, materials, and load characteristics on the mechanical behavior of FGP nanoplates.

3. Research objectives and research areas

3.1. Research objectives

a) Structure

- Plates: Consider FGP nanoplates with different shapes such as rectangular, L-shape, annular, and half-annular.

- Porosity distribution with two rules: evenly distributed and unevenly distributed.

- Elastic foundation: The plate is placed on the Winkler-Pasternak foundation with two layers. The first layer is a spring system with a stiffness coefficient of k_1, while the second layer is a surface layer with a shear stiffness of k_2.

b) Load

Consider FGP nanoplates subjected to static and dynamic loads. Nanostructure devices are mainly subjected to high temperature, moisture, pressure, and pulse load. So in the thesis, in the case of dynamic load, the pulse load is considered.

3.2. Research areas

Studying the mechanical behavior of FGP nanoplates subjected to static and dynamic loads by finite element method based on nonlocal elasticity theory and first-order shear deformation theory.

The thesis has three main problems:

- Static bending problem.
- Free vibration problem.
- Forced vibration problem.

4. Research methods

The finite element method, in combination with nonlocal elasticity theory and Hamilton's principle is used to establish governing equations for the static, free vibration, and forced vibration of FGP nanoplates.

The calculation program is established in the Matlab software. The obtained results are compared with those of other published ones to confirm the correctness of the proposed method.

Thesis structure:

The thesis is organized into an introduction, four chapters, conclusions, and recommendations for future studies as follows:

Introduction: Presenting the urgency of the topic, objectives, objects, areas, and research methods of the thesis.

Chapter 1: Research overview.

Chapter 2: Theoretical basic for calculation of FGP nanoplates resting on an elastic foundation.

Chapter 3: Static analysis of FGP nanoplates resting on an elastic foundation.

Chapter 4: Dynamic analysis of FGP nanoplates resting on an elastic foundation.

Conclusions and recommendations: This chapter summarizes the novel contributions of the thesis and suggests recommendations for future studies.

List of publications

Bibliography

Chapter 1 RESEARCH OVERVIEW

1.1. Overview of nanomaterials and nanostructures

1.1.1. The concept of nanomaterials and nanostructures

Nanomaterials are a type of material with a structure of porosities, fibers, tubes, thin sheets, and so on, with very small sizes ranging from 1 to 100 nanometers. Nanotechnology is a technology related to the design, analysis, calculation, fabrication, and application of nanometer-sized structures and devices.

1.2. Theory deserves nano-size effect

Nowadays, with the development of science and technology, compact devices are required, especially those in medicine, electronics, and aerospace. Nanoplates are one of the important structures commonly used in resonators (Fig. 1.1), sensors, and thin film elements. As a result of their application, understanding the vibration characteristics of the nanoplates is an important issue. Therefore, the vibration analysis of the nanoplates has become a subject of primary interest in recent studies.

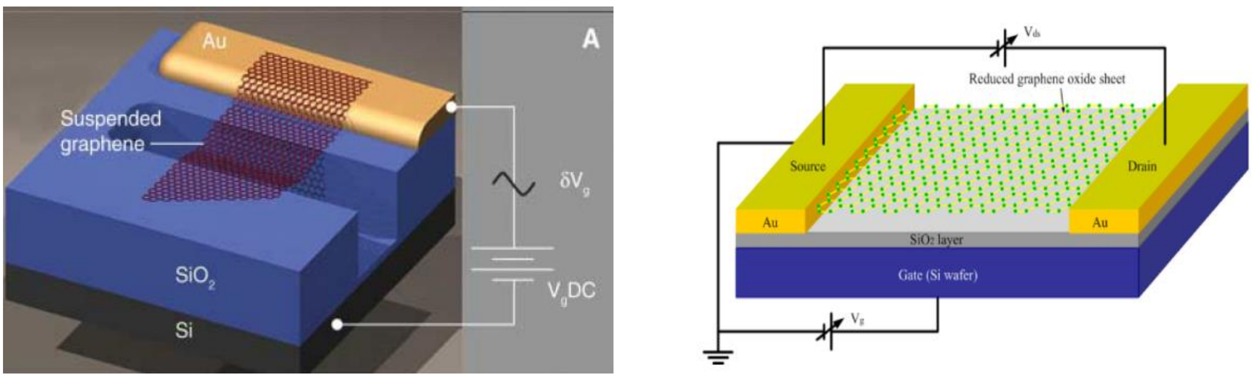


Fig. 1.1. Nanoplates are used in the resonator and sensor.

Theoretical studies and experimental modelings show that the usual calculation theories for structures with sizes of millimeters and above are not accurate for micrometers and nanometers in size. There are three main theories have been proposed to analyze micro/nanostructures, which are the nonlocal elasticity theory, modified couple stress theory, and modified strain gradient theory.

1.2.1. Nonlocal elasticity theory

The nonlocal elasticity theory was initially formulated by Eringen as means of an integral constitutive equation:

$$\sigma_{ij} = \int_{\bar{x}} k(\left|x - \bar{x}\right|, \kappa)\sigma_{ij}^{L} dx \tag{1.1}$$

where σ_{ij} and σ_{ij}^{L} are the components of the nonlocal and local stress tensors, respectively.

k is the kernel function determined in terms of nonlocal parameters κ and neighbor distance $\left|x - \bar{x}\right|$.

$\kappa = e_0 a$ with a and e_0 are the material constant and the parameter depending on the size ratio of the material, respectively. The value e_0 can be determined either from experiments or simulations.

By considering a specific kernel function *k*, Eringen reformulated the nonlocal constitutive equation (1.1) in a differential form as follows:

$$\left(1 - \mu\nabla^2\right)\sigma_{ij} = \sigma_{ij}^{L} \tag{1.2}$$

where $\mu = \kappa^2$, ∇^2 is the Laplace operator.

1.2.2. Modified couple stress theory

The modified couple stress theory was proposed by Yang et al. The strain energy *U* is a function of both strain and curvature as follows

$$U = \frac{1}{2}\int_{V}\left(\sigma_{ij}\varepsilon_{ij} + m_{ij}\chi_{ij}\right)dV \tag{1.3}$$

1.2.3. Modified strain gradient theory

According to the modified couple stress theory, the strain energy is written by

$$U = \frac{1}{2}\int_V \left(\sigma_{ij}\varepsilon_{ij} + p_i\gamma_i + \tau_{ijk}\eta_{ijk} + m_{ij}\chi_{ij}\right)dV \qquad (1.4)$$

1.3. The main contents of the thesis

From the above conclusions, the thesis combines the finite element method, the nonlocal elasticity theory, and the first-order shear deformation theory to analyze static bending, free vibration, and forced vibration of nanoplates. In which the structure rests on an elastic foundation with different shapes such as rectangular, L-shape, annular, and half-annular. Two types of porosity distribution are considered, which are evenly distributed and unevenly distributed.

The results of the thesis are compared with the published results by analytical solutions to verify the accuracy and reliability of the calculation method and program. Then, the thesis will investigate some geometrical and material properties, elastic foundation, boundary conditions to the static bending, free vibration, and forced vibration of nanoplates.

Chapter 2 THEORETICAL BASIC FOR CALCULATION OF THE FGP NANOPLATES RESTING ON ELASTIC FOUNDATION

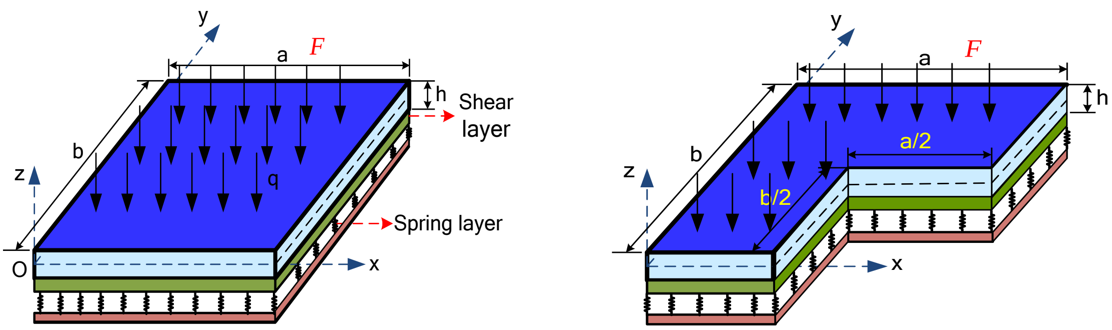
2.1. Material and mechanical models and assumptions

2.1.1. Material and mechanical models

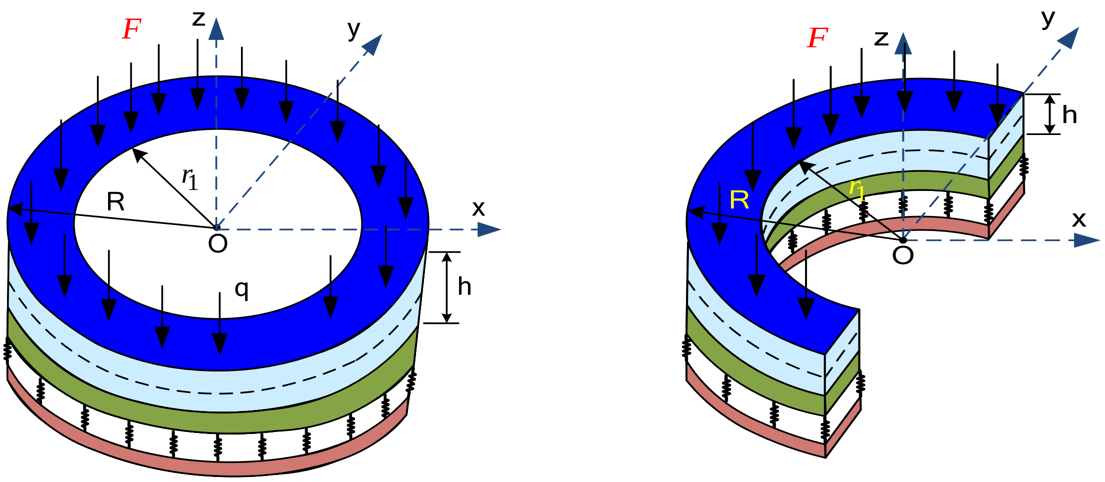
Consider FGP nanoplates with different shapes, such as rectangular, L-shape, annular, and half-annular. The plate is placed on a two-coefficient Winkler-Pasternak elastic foundation with two continuous layers. The first layer is a parallel spring system with a stiffness coefficient k_1, while the second layer is the shear layer with stiffness coefficient k_2. The porosity distribution in nanoplates according to two rules as evenly distributed and unevenly distributed (Fig. 2.1). The plate is subjected to static loads and pulsating dynamic loads, which are perpendicular to the plate surface (Fig. 2.2).

a) Even porosity *b) Uneven porosity*

Fig. 2.1. Two cases of porosities.



a) Rectangular nanoplate *b) L-shape nanoplate*



c) Annular nanoplate *d) Half-annular nanoplate*

Fig. 2.2. The model of FGP nanoplates resting on the elastic foundation.

2.1.2. Assumptions

To establish the mechanical behaviour relations, the thesis uses some assumptions as follows:

- The plate satisfies the Mindlin plate theory with $\varepsilon_z = 0$.
- Displacement of structures is small.
- Material is linear elastic.

2.2. Mechanical properties of materials

The FGP materials with the variation of two constituents and two different distributions of porosity through thickness are determined as follows:

Case 1: Even porosity $P(z) = P_m + (P_c - P_m)\left(\frac{z}{h} + 0.5\right)^k - \frac{\xi}{2}(P_c + P_m)$ (2.1)

Case 2: Uneven porosity: $P(z) = P_m + (P_c - P_m)\left(\frac{z}{h} + 0.5\right)^k - \frac{\xi}{2}(P_c + P_m)\left(1 - \frac{2|z|}{h}\right)$ (2.2)

where *P* represents material properties such as the modulus of elasticity *E*, mass density ρ, and Poisson's ratio ν; $k\ (k > 0)$ is the power-law index; $\xi\ (0 \le \xi \le 0.5)$ is porosity factor. The symbols *m* and *c* represent metal and ceramic components, respectively.

2.3. The mechanical behavior relations of the plate

2.3.1. The displacement field

Based on the first-order shear deformation theory, the displacement field of the FGP nanoplates is defined by

$$\begin{cases} u(x,y,z) = u_0(x,y) + z\varphi_x(x,y) \\ v(x,y,z) = v_0(x,y) + z\varphi_y(x,y) \\ w(x,y,z) = w_0(x,y) \end{cases} \tag{2.3}$$

2.3.2. Strain – displacement relations

The strain vector of the plate is deduced from the displacement field as follows: $\boldsymbol{\varepsilon} = \{\boldsymbol{\varepsilon}_1 \quad \boldsymbol{\varepsilon}_2\}^T = \{\boldsymbol{\varepsilon}_1^0 + z\boldsymbol{\varepsilon}_1^1 \quad \boldsymbol{\varepsilon}_2^0\}^T$ (2.4)

2.3.3. Stress-strain relations

According to the nonlocal elasticity theory, the stress-strain relation is determined by

$$\boldsymbol{\sigma} - \mu\nabla^2\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\varepsilon} \tag{2.4}$$

in which $\mu = (e_0 a)^2$ is nonlocal factor, which represents the small-scale effect in nanostructures, e_0 is a constant, a is an internal characteristic length, and $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the Laplace operator.

2.3.4. Hamilton principle

To obtain the governing equations of motion of FGP nanoplates, the Hamilton's principle is applied in the form as follows:

$$\int_0^t (\delta U + \delta V - \delta W - \delta T)\,dt = 0 \tag{2.5}$$

The variation of the strain energy can be given by:

$$\delta U = \int_S \int_{-h/2}^{h/2} \left(\sigma_{xx}\delta\varepsilon_{xx} + \sigma_{yy}\delta\varepsilon_{yy} + \sigma_{xy}\delta\varepsilon_{xy} + \sigma_{xz}\delta\varepsilon_{xz} + \sigma_{yz}\delta\varepsilon_{yz} \right) dzdS \tag{2.6}$$

The variation of the energy stored in the deformed elastic foundation is expressed by:

$$\delta V = \int_S \left(k_1.w - k_2\nabla^2 w \right)\delta w dS = \int_S \left(k_1.w_0 - k_2\nabla^2 w_0 \right)\delta w_0 dS \tag{2.7}$$

The variation of kinetic energy is given by

$$\delta T = \int_S \int_{-h/2}^{h/2} \rho(z) \begin{pmatrix} \dot{u}_0\delta\dot{u}_0 + \dot{u}_0 z\delta\dot{\varphi}_x + z\dot{\varphi}_x\delta\dot{u}_0 + z^2\dot{\varphi}_x\delta\dot{\varphi}_x \\ +\dot{v}_0\delta\dot{v}_0 + \dot{v}_0 z\delta\dot{\varphi}_y + z\dot{\varphi}_y\delta\dot{v}_0 + z^2\dot{\varphi}_y\delta\dot{\varphi}_y + \dot{w}_0\delta\dot{w}_0 \end{pmatrix} dzdS \tag{2.8}$$

The variation of work done by applied force is expressed by:

$$\delta W = \int_S F\delta w dS = \int_S F\delta w_0 dS \tag{2.9}$$

2.4. Finite element formulations

2.4.1. Finite element model

In this thesis, the eight-node rectangular element, which consisted of four nodes at the vertices of the quadrilateral and four nodes being the midpoints of the element's edge, is used. Each node has five degrees of freedom $\{u_0 \quad v_0 \quad w_0 \quad \varphi_x \quad \varphi_y\}$.

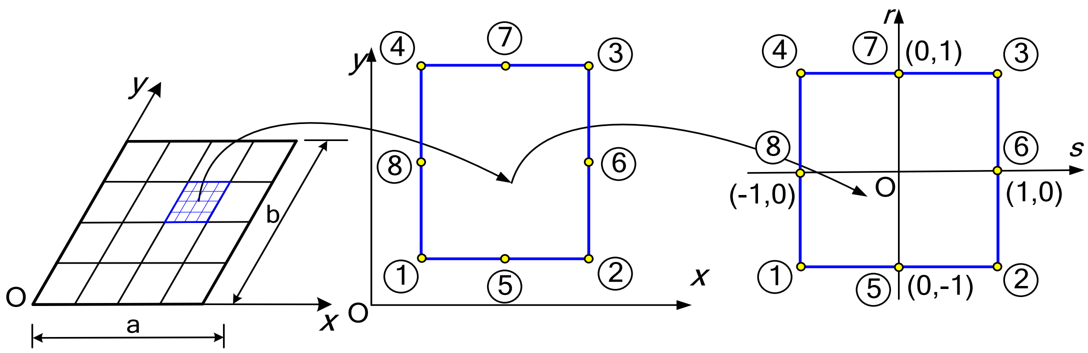


Fig. 2.3. The eight-node rectangular element.

2.4.2. Element matrices and element vectors

The node displacement vector is expressed by

$$\underset{5\times1}{\mathbf{q}_i} = \{u_{0i} \quad v_{0i} \quad w_{0i} \quad \varphi_{xi} \quad \varphi_{yi}\}^T \tag{2.10}$$

The element displacement vector is

$$\underset{40\times1}{\mathbf{q}_e} = \left[\mathbf{q}_1^T \quad \mathbf{q}_2^T \quad \mathbf{q}_3^T \quad \mathbf{q}_4^T \quad \mathbf{q}_5^T \quad \mathbf{q}_6^T \quad \mathbf{q}_7^T \quad \mathbf{q}_8^T\right]^T \tag{2.11}$$

The element stiffness matrix is defined by

$$\mathbf{K}_e^b = \int_{-1}^{1}\int_{-1}^{1}\left(\begin{bmatrix}\mathbf{B}_1^T & \mathbf{B}_2^T\end{bmatrix}\begin{bmatrix}\mathbf{A} & \mathbf{B}\\ \mathbf{B} & \mathbf{X}\end{bmatrix}\begin{bmatrix}\mathbf{B}_1\\ \mathbf{B}_2\end{bmatrix}\right)\det|J|drds \tag{2.12}$$

$$\mathbf{K}_e^s = \int_{-1}^{1}\int_{-1}^{1}\left(\mathbf{B}_3^T\mathbf{A}^s\mathbf{B}_3\right)\det|J|drds \tag{2.13}$$

The elastic foundation stiffness matrix is determined as follows:

$$\mathbf{K}_e^f = \int_{-1}^{1}\int_{-1}^{1}\left\{\begin{array}{l} k_1\left[\mathbf{N}_w^T\mathbf{N}_w + \mu\left(\dfrac{\partial \mathbf{N}_w^T}{\partial x}\dfrac{\partial \mathbf{N}_w}{\partial x} + \dfrac{\partial \mathbf{N}_w^T}{\partial y}\dfrac{\partial \mathbf{N}_w}{\partial y}\right)\right] \\ +k_2\left[\begin{array}{l}\left(\dfrac{\partial \mathbf{N}_w^T}{\partial x}\dfrac{\partial \mathbf{N}_w}{\partial x} + \dfrac{\partial \mathbf{N}_w^T}{\partial y}\dfrac{\partial \mathbf{N}_w}{\partial y}\right) \\ +\mu\left(\begin{array}{l}\dfrac{\partial^2 \mathbf{N}_w^T}{\partial x^2}\dfrac{\partial^2 \mathbf{N}_w}{\partial x^2} + \dfrac{\partial^2 \mathbf{N}_w^T}{\partial y^2}\dfrac{\partial^2 \mathbf{N}_w}{\partial y^2} \\ \dfrac{\partial^2 \mathbf{N}_w^T}{\partial x^2}\dfrac{\partial^2 \mathbf{N}_w}{\partial y^2} + \dfrac{\partial^2 \mathbf{N}_w^T}{\partial y^2}\dfrac{\partial^2 \mathbf{N}_w}{\partial x^2}\end{array}\right)\end{array}\right]\end{array}\right\}\det|J|drds \tag{2.14}$$

The element mass matrix is :

$$\mathbf{M}_e = \int_{-1}^{1}\int_{-1}^{1}\left[\mathbf{N}^T\mathbf{D}_m\mathbf{N} + \mu\left(\frac{\partial \mathbf{N}^T}{\partial x}\mathbf{D}_m\frac{\partial \mathbf{N}}{\partial x} + \frac{\partial \mathbf{N}^T}{\partial y}\mathbf{D}_m\frac{\partial \mathbf{N}}{\partial y}\right)\right]\det\left|J\right|drds \tag{2.15}$$

The element load vector is defined by

$$\mathbf{F}_{\mathrm{e}} = \int_{-1}^{1}\int_{-1}^{1}\left\{\mathbf{N}_w^T F - \mu\left[F\left(\frac{\partial^2 \mathbf{N}_w^T}{\partial x^2} + \frac{\partial^2 \mathbf{N}_w^T}{\partial y^2}\right) + \mathbf{N}_w^T\left(\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2}\right)\right]\right\}\det\left|J\right|drds \tag{2.16}$$

All integrations in equations (2.12), (2.13), (2.14), (2.15), and (2.16) are calculated using full Gauss integration with three integral points.

The governing equation of motion of the plate element without resistance has the form:

$$\mathbf{M}_e.\ddot{\mathbf{q}}_e + \mathbf{K}_e.\mathbf{q}_e = \mathbf{F}_e \tag{2.17}$$

The governing equation of motion of the plate without resistance as follows:

$$\mathbf{M}.\ddot{\mathbf{q}} + \mathbf{K}.\mathbf{q} = \mathbf{F} \tag{2.18}$$

2.5. Summary of Chapter 2

Using the finite element method based on first-order shear deformation theory and nonlocal elasticity theory, the author has built a stiffness matrix, mass matrix, and load vector. From there, based on Hamilton's principle, the motion equation is established for calculating FGP nanoplates with different shapes resting on elastic foundation.

Defining specific boundary conditions for different shapes nanoplates.

The formulations in this chapter are the scientific basis for building algorithms and calculation programs to solve the static problem in chapter 3, the free vibration problem, and forced vibration in chapter 4.

The expanded expressions in this chapter are used for calculations in the next studies.

Chapter 3 STATIC ANALYSIS OF FGP NANOPLATES RESTING ON ELASTIC FOUNDATION

3.1. Finite element algorithm and calculation programs

The equilibrium equation of nanoplates subjected to static loads is:

$$\mathbf{K}.\mathbf{q} = \mathbf{F} \tag{3.1}$$

From the equation (3.1), the global node displacement vector is calculated according to the following formulation:

$$\mathbf{q} = \mathbf{K}^{-1}\mathbf{F} \tag{3.2}$$

The program FGP_Nanoplates_FSDT_Nonlocal_Static_2022 (FNFNS_2022) is built in Matlab software to analyze the static behavior of the FGP nanoplates resting on the elastic foundation.

3.2. Verification study

Considering completely simply supported FGM square nanoplates. Material properties of the individual materials as shown in Table 3.1, and geometry parameters as $a=b=10\ nm$, $h=a/10$, $k=0$, $K_2=0$.

The dimensionless quantities are introduced by

$$w^* = \frac{100E_2h^3}{q_0a^4}w, \sigma_{xx}^* = \frac{10h}{q_0a}\sigma_{xx}, \sigma_{xy}^* = \frac{10h}{q_0a}\sigma_{xy}, K_1 = \frac{k_wa^4}{H}, K_2 = \frac{k_sa^2}{H}, H = \frac{E_2h^3}{12(1-\nu^2)} \tag{3.3}$$

Table 3.1. Material properties

Materials	E (GPa)	ν	ρ (kg/m^3)
Al_2O_3 (ceramic)	380	0.3	3800
Al (metal)	70	0.3	2707

As exhibited in Table 3.2 the present results are in good agreement with an analytical method of Sobhy. It means that the present method is highly reliable.

Table 3.2. The displacement and stress of square nanoplates.

Method	K_1	$\mu=0$		$\mu=4$	
		$w^*\left(\frac{a}{2};\frac{b}{2};0\right)$	$\sigma_{xx}^*\left(\frac{a}{2};\frac{b}{2};\frac{h}{2}\right)$	$w^*\left(\frac{a}{2};\frac{b}{2};0\right)$	$\sigma_{xx}^*\left(\frac{a}{2};\frac{b}{2};\frac{h}{2}\right)$
Sobhy	0	2.9603	19.9550	5.2977	35.7108
	100	2.3290	15.6991	3.5671	24.0455
Thesis	0	2.9600 *(0.01%)*	19.8990 *(0.28%)*	5.2971 *(0.01%)*	35.6106 *(0.28%)*
	100	2.3288 *(0.01%)*	15.6555 *(4.36%)*	3.5669 *(0.01%)*	23.9791 *(0.28%)*

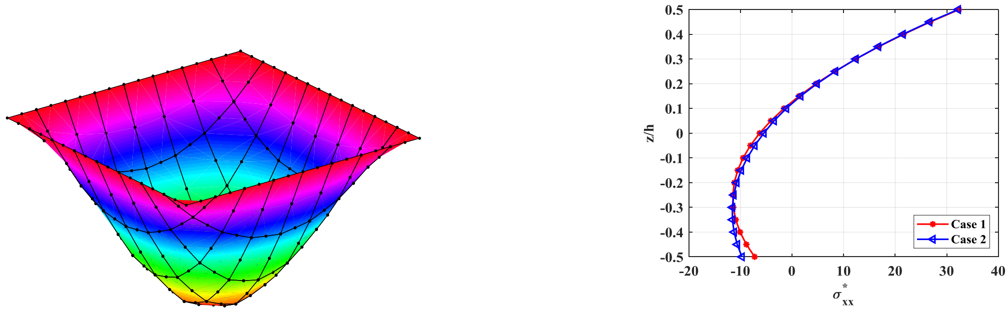
3.3. Numerical results and discussion

Based on the calculation program, in this section, the author analyzes the static bending of FGP nanoplates with different shapes, geometrical parameters, material properties, boundary conditions, and elastic stiffness. The static load is evenly distributed in the direction perpendicular to the plate surface.

3.3.1. Rectangular nanoplate

Considering the CSS FGP square nanoplates with geometric dimensions $a=b=10nm$; $h=a/10$. Material properties of the individual materials as shown in Table 3.1, power-law index k=1, porosity factor $\xi=0.1$, nonlocal factor $\mu=2$, the stiffness of foundation: K_1=50, K_2=10. The plate is subjected to uniform load q_0 in perpendicular directions.

The deformation field and stress of the nanoplate are shown in Figure 3.1. It can be seen that the law of stress distribution according to the thickness of the plate at a point is consistent with the law of effective mechanical properties of FGP materials. In addition, in the case of a square plate, with completely simply supported, the maximum displacement will be at the center of the plate, and the strain field varies uniformly from this point to the surrounding.

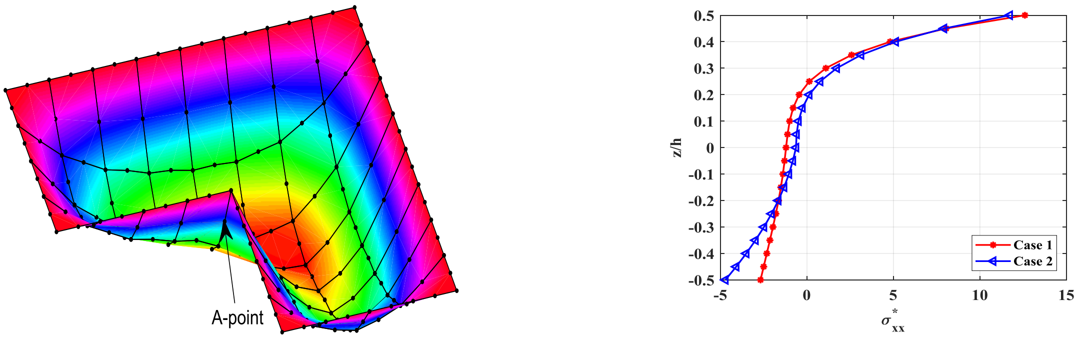


a) The deformation field *b) The stress σ^*_{xx} at the midpoint*

Figure 3.1. The deformation and stress of the CSS square nanoplate.

3.3.2. L-shape nanoplate

The deformation field and stress of the L-shape nanoplate is shown in Figure 3.2. From the deformation field can be seen that the vicinity of the L-angle is most susceptible to failure due to stress concentration because of the sudden change in shape at the L-angle.

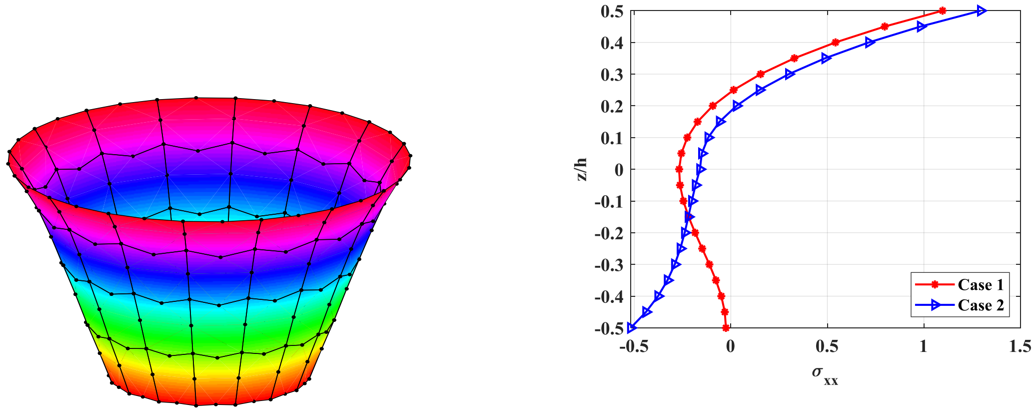


a) The deformation field *b) The stress σ^*_{xx} at A-point*

Figure 3.2. The deformation and stress of the CSS L-shape nanoplate.

3.3.3. Annular nanoplate

The deformation field and stress of the annular nanoplate are indicated in Figure 3.3.

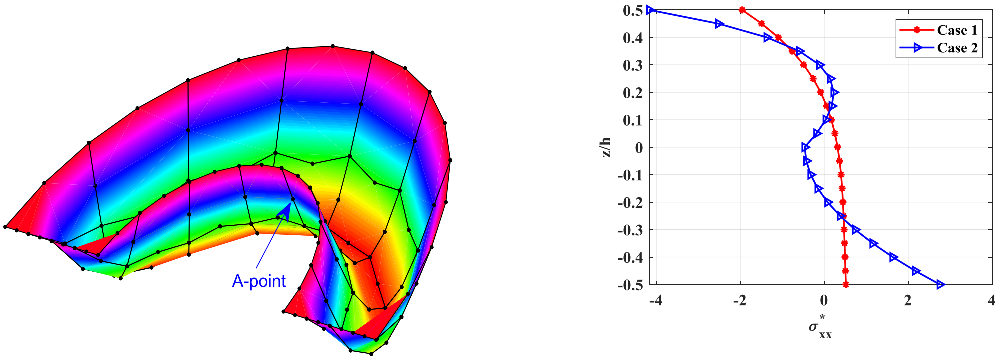


a) The deformation field *b) The stress σ^*_{xx} at the A-point*

Figure 3.3. The deformation and stresses of the clamped supported at the outer border FGP annular nanoplate

3.3.4. Half-annular nanoplate

The deformation field and stress of the FGP annular nanoplate are indicated in Figure 3.4.



a) The deformation field *b) The stress σ^*_{xx} at the A-point*

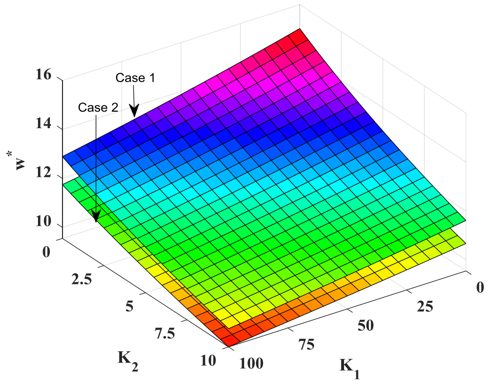
Figure 3.4. The deformation and stresses of the clamped supported at outer border FGP half-annular nanoplate.

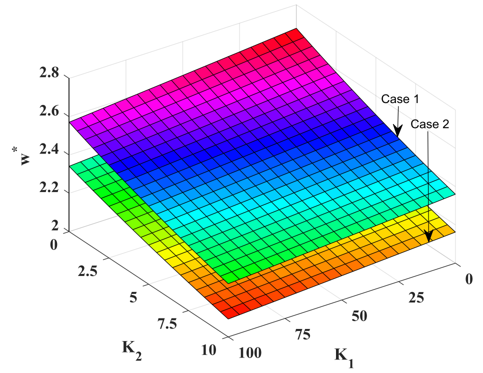
3.4. The influence of some factors on the static response of FGP nanoplates

3.4.1. The influence of the parameters of the elastic foundation

From the numerical results shown in Fig. 3.5, it can be concluded that when increasing K_1 and K_2 leads to the displacement of nanoplates decrease. This is perfectly reasonable because when the foundation stiffness increases, the stiffness of the plate increase. Furthermore, the

Pasternak foundation supports more strongly than the Winkler foundation. In other words, the shear layer of the foundation provides better support than the spring layer.





a) CSS square nanoplate *b) CSS L-shape nanoplate*

Fig. 3.5. Effect of K_1 and K_2 on the displacement of FGP nanoplates.

3.5. Summary of Chapter 3

In this chapter, the author presents the algorithm to analyze the FGP nanoplates with different shapes resting on an elastic foundation subjected to static loads. From the proposed formulation and the numerical results, the author can withdraw some following points:

The author has built an algorithm and a calculation program FGP_Nanoplates_FSDT_Nonlocal_Static_2022 (FNFNS_2022) to calculate the FGP nanoplates resting on an elastic foundation under static load. The calculation results of the program are compared with other published results showing accuracy and reliability.

The survey results show that there are many factors affecting the static response of FGP nanoplates resting on the elastic foundation. However, there are significant influencing factors, such as the nonlocal factor, the parameters of FGP, and the foundation stiffness. Therefore, when designing nanoplates for special requirements, engineers must attention to the above issues for the structure to operate at high efficiency.

The obtained results in this chapter have been shown in paper number 2 and number 4 (List of publications).

Chapter 4 DYNAMIC ANALYSIS OF FGP NANOPLATES RESTING ON ELASTIC FOUNDATION

4.1. Free vibration problem

4.1.1. Finite element algorithm and calculation programs

From the equation of motion, in case of the external load is zero, the free vibration equation of the plate is as follows:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{0} \tag{4.1}$$

Assuming that the oscillations are harmonic with amplitude $\mathbf{q}_0$ and frequency ω, then the solution of the equation (4.1) has form $\mathbf{q} = \mathbf{q}_0 \sin(\omega \mathrm{t})$. From there, the free vibration equation (4.1) leads to:

$$\left(\mathbf{K} - \mathbf{M}\omega^2\right)\mathbf{q}_0 = \mathbf{0} \tag{4.2}$$

The equations (4.2) are homogeneous linear equations. It has a non-trivial solution $\mathbf{q}_0 \neq \mathbf{0}$ if and only if:

$$\left\|\left(\mathbf{K} - \mathbf{M}\omega^2\right)\right\| = 0 \tag{4.3}$$

The equation (4.3) is a polynomial equation of order N. Solving this equation give N values of natural frequencies ω_i. Substituting natural frequencies ω_i into the equation (4.2) we find the corresponding eigenvector.

The program FGP_Nanoplates_FSDT_Nonlocal_Freevibration_2022 (FNFNF_2022) is developed in Matlab software to analyze the free vibrations of FGP nanoplates resting on an elastic foundation.

4.1.2. Convergence and verification study

The CSS homogeneous square nanoplate with geometrical parameters *a* = *b* = 10 *nm*, *h* = *a*/10 and material properties: *E = 30MPa*, $\nu = 0.3$ is studied. The dimensionless natural frequencies $\omega^* = \omega h\sqrt{\rho / \mathrm{G}}$ are listed in Table 4.1. It can be seen that the obtained results in the thesis are closed to those of Belkorissat et al. based on hyperbolic refined plate theory (the maximum error is 1.18 %) and results of Aghababaei et al. using Navier solution based on the first-order shear deformation theory and the third-order shear deformation theory (the maximum error is less than 1%). Numerical results also indicate that the obtained frequencies based on the classical plate theory are significantly greater than the obtained results using other theories, the maximum error is 3 % in the case of moderately thick plates.

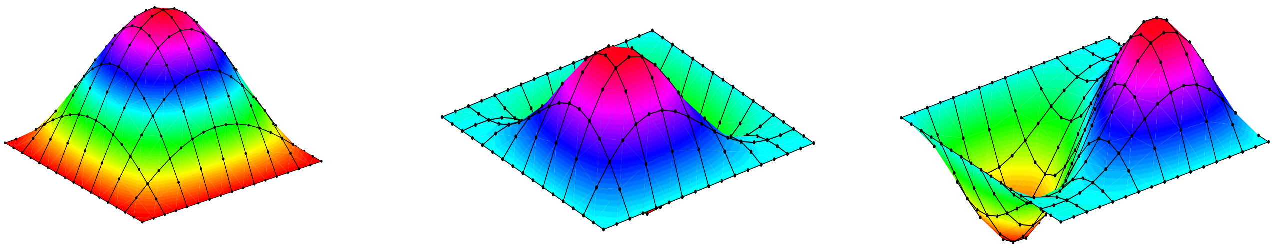
Table 4.1. The first dimensionless natural frequency of homogeneous square nanoplates.

a/h	μ	HSDT Belkorissat et al.		CPT Aghababaei et al.		FSDT Aghababaei et al.		TSDT Aghababaei et al.		Thesis
		ω_1^*	*Er*	ω_1^*	*Er*	ω_1^*	*Er*	ω_1^*	*Er*	
10	0	0.0930	*0.32*	0.0936	*0.32*	0.0930	*0.32*	0.0935	*0.21*	0.0933
	1	0.0850	*0.35*	0.0880	*3.07*	0.0850	*0.35*	0.0854	*0.12*	0.0853
	2	0.0787	*0.51*	0.0816	*3.06*	0.0788	*0.38*	0.0791	*0.00*	0.0791
	3	0.0737	*0.14*	0.0763	*3.28*	0.0737	*0.14*	0.0741	*0.40*	0.0738
	4	0.0695	*0.29*	0.0720	*3.19*	0.0696	*0.14*	0.0699	*0.29*	0.0697
	5	0.0659	*0.30*	0.0683	*3.22*	0.0660	*0.15*	0.0663	*0.30*	0.0661
20	0	0.0238	*0.00*	0.0239	*0.42*	0.0239	*0.42*	0.0238	*0.00*	0.0238
	1	0.0218	*0.00*	0.0220	*0.91*	0.0218	*0.00*	0.0218	*0.00*	0.0218
	2	0.0202	*0.50*	0.0204	*0.49*	0.0202	*0.50*	0.0202	*0.50*	0.0203
	3	0.0189	*0.53*	0.0191	*0.52*	0.0189	*0.53*	0.0189	*0.53*	0.0190
	4	0.0178	*0.56*	0.0180	*0.56*	0.0179	*0.00*	0.0179	*0.00*	0.0179
	5	0.0169	*1.18*	0.0171	*0.00*	0.0170	*0.59*	0.0170	*0.59*	0.0171

4.1.3. Numerical results and discussion

4.1.3.1. Square nanoplate

Fig. 4.1 show the first three mode shapes of FGP square nanoplate, with geometric dimensions a = b =10 nm, h = a/10, $K_1 = 100$, $K_2 = 10$, $k = 1$, $\mu = 2$, $\xi = 0.2$. It can be observed that the second dimensionless frequency is equal to the third dimensionless frequency. This is suitable for symmetrical nanoplates under the same supported conditions.

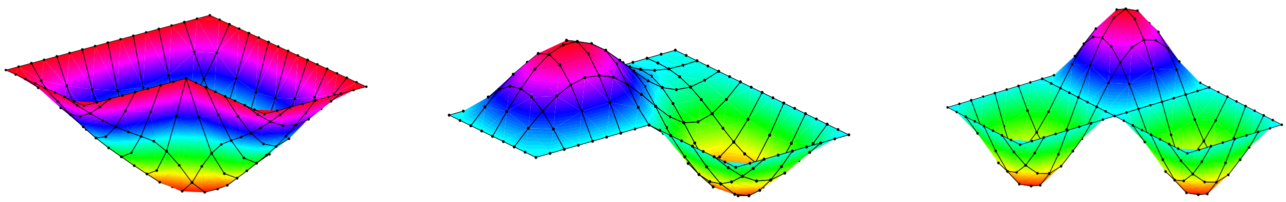


a) 1st mode, Ω_1 = 0.8442 *b) 2nd mode, Ω_2 = 1.5156* *c) 3rd mode, Ω_3 = 1.5156*

Fig. 4.1. The first three mode shapes of the CSS FGP square nanoplate.

4.1.3.2. L-shape nanoplate

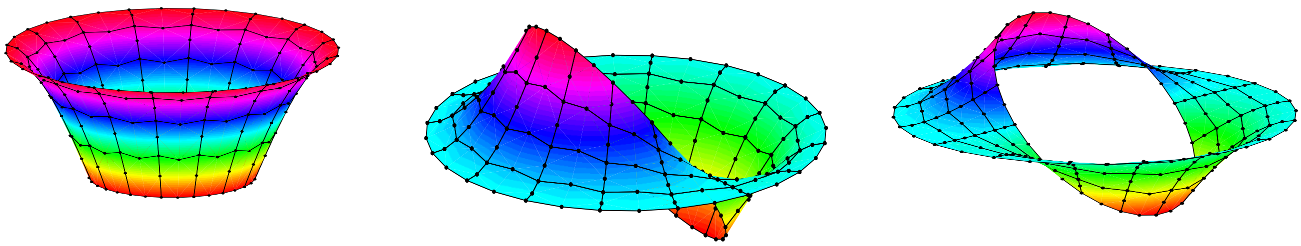
The first three mode shapes of L-shape nanoplate are shown in Fig. 4.2.



a) 1st mode, Ω_1 =1.5102 *b) 2nd mode, Ω_2 =1.7367* *c) 3rd mode, Ω_3 =1.9983*

Fig. 4.2. The first three mode shapes of the CSS FGP L-shape nanoplate.

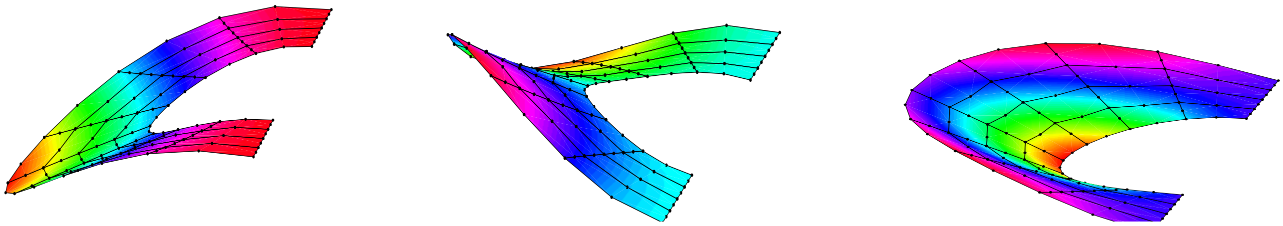
4.1.3.3. Annular nanoplate



a) 1st mode, Ω_1 =1.7710 *b) 2nd mode, Ω_2 =1.9901* *c) 3rd mode, Ω_3 =1.9901*

Fig. 4.3. The first three mode shapes of the CCS FGP annular nanoplate.

4.1.3.4. Half-annular nanoplate



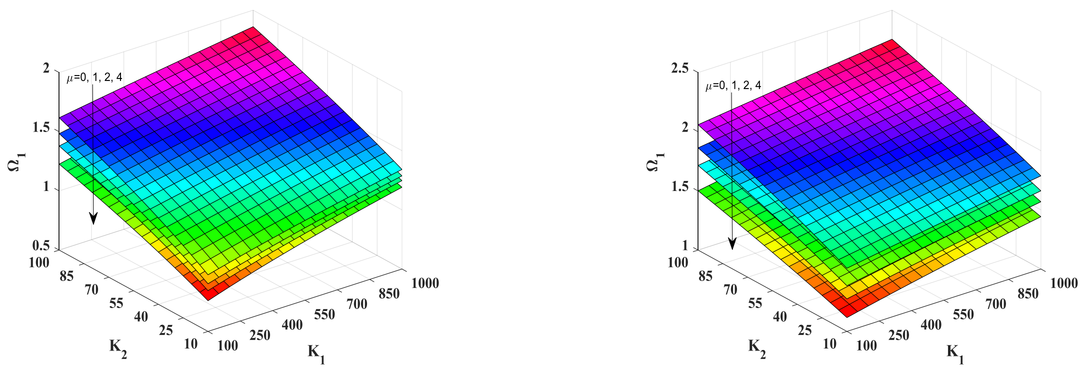
a) 1^{st} mode, Ω_1 =0.6367 *b) 2^{nd} mode, Ω_2 =1.0892* *c)3^{rd} mode, Ω_3 =1.3944*

Fig. 4.4. The first three mode shapes of the CCS FGP half-annular nanoplate.

4.1.4. Influence of some factors on the natural frequency of FGP nanoplates

4.1.4.1. Influence of the elastic foundation

In order to consider the influences of foundation stiffness on free vibration of the nanoplate, change K_1 from 100 to 1000 and K_2 from 10 to 100. The first natural frequencies of the nanoplate are shown in Fig. 4.5. It can be found that when K_1 and K_2 increase lead to the natural frequency of nanoplates increase. Furthermore, the effects of the Pasternak foundation are stronger than the Winkler foundation.



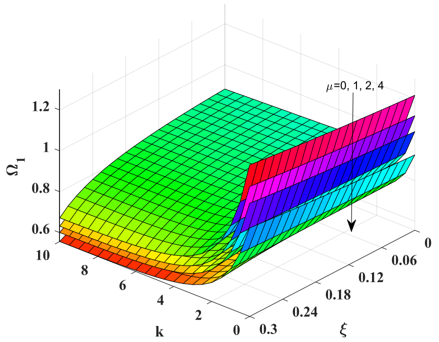
a) The CSS FGP nanoplate *b) The CCS FGP nanoplate*

Fig. 4.5. Natural frequencies of FGP square nanoplate versus K_1 and K_2.

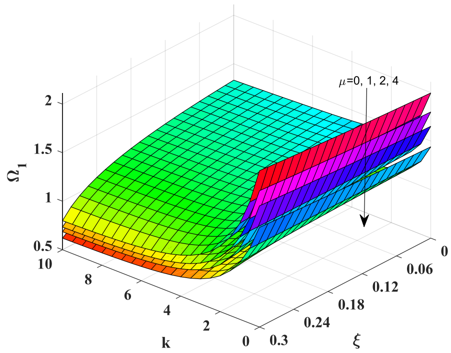
4.1.4.2. Influence of material properties

Secondly, let's consider the effect of material properties on the free vibration of the FGP square nanoplate. The power-law index k gets values from 0 to 10, and the porosity factor changes from 0 to 0.3, the foundation stiffness $K_1 = 100$, $K_2 = 10$, and nonlocal factor $\mu = 0, 1, 2, 4$. The authors only choose the power-law index in the range from 0 to 10 for investigation because many published works show that when k is greater than 10, the natural frequency of FGP structures does not change much and the recommended value of porosity volume fraction is in the range (0-0.3).

The natural frequencies of FGP nanoplate with different boundary conditions are listed in Fig. 4.6. It can be seen that when k increases, the stiffness of the FGP nanoplate decreases (nanoplate is metal-rich), and hence natural frequencies decrease. We also found that when the increase of nonlocal factor leads to natural frequencies of FGP nanoplates decrease. The results are quite reasonable because the increase of the nonlocal factor reduces the stiffness of structures in the nonlocal elastic theory.



a) The CSS FGP nanoplate



b) The CCS FGP nanoplate

Fig. 4.6. Natural frequencies of FGP square nanoplate versus k and ξ.

4.2. Forced vibration problem

4.2.1. Finite element algorithm and calculation programs

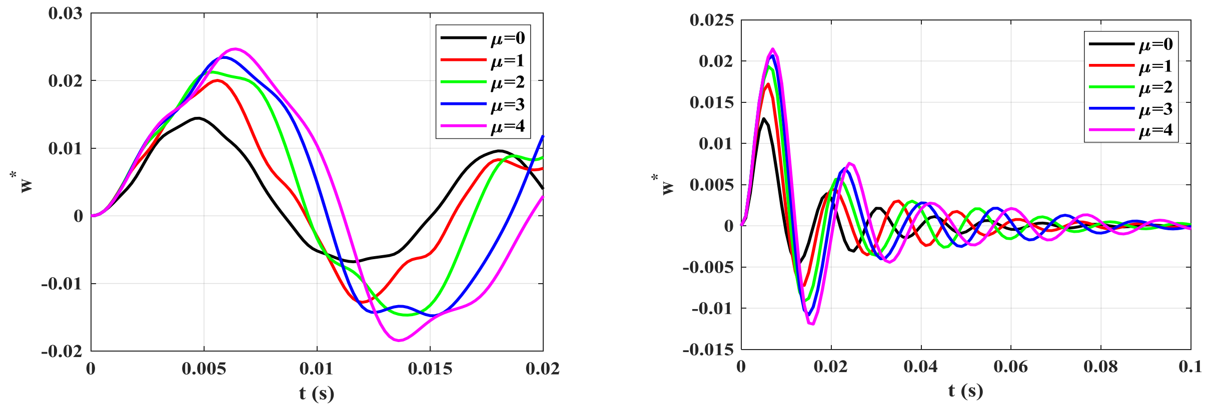
Considering the drag coefficient, which is linearly dependent on velocity, the governing equations of motion for forced vibration of the plate have the following form:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{F} \tag{4.4}$$

To solve the system of differential equations (4.4), the thesis uses the Newmark method of direct integration.

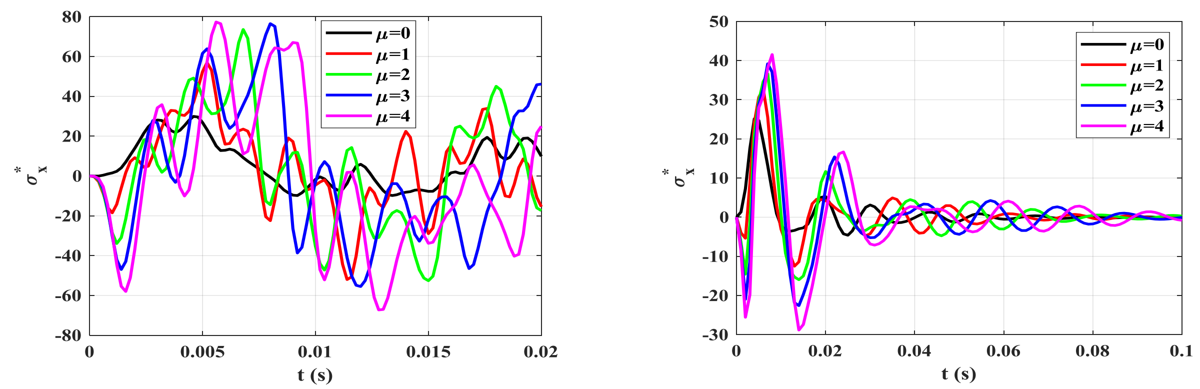
4.2.2. Numerical results and discussion

Fig. 4.7 – Fig. 4.8. The stress response σ_{xx}^{*} of the A-point over time t at z = $h/2$. Fig. 4.8 present the effect of the nonlocal factor on the displacement and stress response of the completely simply supported FGP L-shape nanoplate with uneven porosity at A-point has coordinates (3.75, 6.25) in two cases: without damping ratio $\zeta = 0$ and include damping ratio $\zeta = 0.1$. It can be seen that the vibration is damped after the application time of the load, and the nonlocal factor reduces the stiffness of nanoplates; hence displacement and stress increase.



a) The damping ratio $\zeta = 0$ *b) The damping ratio* $\zeta = 0.1$

Fig. 4.7. The deflection response of the A-point over time *t*.



a) The damping ratio $\zeta = 0$ *b) The damping ratio* $\zeta = 0.1$

Fig. 4.8. The stress response σ_{xx}^{*} of the A-point over time t at z = $h/2$.

4.3. Summary of Chapter 4

In this chapter, the author presented the free vibration problem and forced vibration problem. From the proposed formulation and the numerical results, the author can withdraw some following points:

The author has built an algorithm and a calculation program FGP_Nanoplates_FSDT_Nonlocal_Freevibration_2022 (FNFNF_2022) to calculate the free vibration of the FGP nanoplates resting on an elastic foundation. The calculation results of the program are compared with other published results showing accuracy and reliability.

The author has built an algorithm and a calculation program FGP_Nanoplates_FSDT_Nonlocal_Dynamic_2022 (FNFND_2022) to calculate the FGP nanoplates resting on an elastic foundation under dynamic load. The calculation results of the program are compared with other published results showing accuracy and reliability.

The survey results show that there are many factors affecting the free vibration of FGP nanoplates resting on an elastic foundation. However, there are large influencing factors such as nonlocal factor, the parameters of the functionally graded porous material, stiffness of the elastic foundation

The survey results show that there are many factors affecting the dynamic response of FGP nanoplates resting on an elastic foundation. However, there are large influencing factors, such as the nonlocal factor, the parameters of the functionally graded porous material, and the stiffness of the elastic foundation. Therefore, when designing nanoplates for special requirements, engineers need to pay attention to the above issues for the structure to operate at high efficiency.

The obtained results in this chapter have been shown in papers number 1, number 3, and number 5 (List of publications).

CONCLUSIONS
AND RECOMMENDATIONS FOR FUTURE STUDIES

1. Novel contributions of the thesis

Based on the finite element method and nonlocal elastic theory, the static and dynamic responses of FGP nanoplates resting on an elastic foundation have been investigated in this thesis. The following are some of the significant contributions of the thesis:

- The thesis established a model, a finite element algorithm, and a collection of programs to analyze of static bending, free and forced vibration of the FGP nanoplates resting on an elastic foundation with various plate shapes and boundary conditions. The findings demonstrate the difference between nonlocal elasticity theory and local elasticity theory.

- The influence of parameters such as nonlocal coefficients, material properties, geometric dimensions, elastic foundation stiffness, etc., on the static response, and natural and forced vibrations of FGP nanoplates have been examined in this thesis. From there, the thesis provides scientifically and practically relevant commentary.

- The data set of the dissertation may be used as a reference in the computation and design of nanostructures to handle static and dynamic loads encountered in sensors, electronic chips, and sensors.

2. Recommendations for future studies

- Using higher-order shear deformation theories to examine the vibrations of nanoplates and nanoshells subjected to various types of mechanical loads while taking temperature into consideration.

- Investigating the buckling problems of nanoplates in a viscoelastic environment while subjected to a variety of mechanical loads.

- Computing the shape optimization problem and material optimization issue for nanoplates.

- Calculation of nanomaterial-reinforced structures exposed to various sorts of loads.